

Foundations for programming languages 5: An answer to the exercise 4

Exercise 4 Derive the state after executing the statement `while(Y){Y=(Y-20);}` in the state $\sigma = \{(X, 10), (Y, 40), (Z, 30)\}$.

(An answer)

$$\frac{\frac{\langle Y, \sigma \rangle \rightarrow 40 \quad \langle 20, \sigma \rangle \rightarrow 20}{\langle (Y-20), \sigma \rangle \rightarrow 20} \quad (*)}{\frac{\langle Y, \sigma \rangle \rightarrow 40 \quad \langle Y=(Y-20);, \sigma \rangle \rightarrow \sigma[20/Y] \quad \langle \text{while}(Y)\{Y=(Y-20);\}, \sigma[20/Y] \rangle \rightarrow \sigma[20/Y][0/Y]}{\langle \text{while}(Y)\{Y=(Y-20);\}, \sigma \rangle \rightarrow \sigma[20/Y][0/Y]}}$$

We write the part (*) with the relation below the line.

$$\frac{\frac{\langle Y, \sigma[20/Y] \rangle \rightarrow 20 \quad \langle 20, \sigma[20/Y] \rangle \rightarrow 20}{\langle (Y-20), \sigma[20/Y] \rangle \rightarrow 0} \quad \langle Y, \sigma[20/Y][0/Y] \rangle \rightarrow 0}{\frac{\langle Y, \sigma[20/Y] \rangle \rightarrow 20 \quad \langle Y=(Y-20);, \sigma[20/Y] \rangle \rightarrow \sigma[20/Y][0/Y] \quad \langle \text{while}(Y)\{Y=(Y-20);\}, \sigma[20/Y][0/Y] \rangle \rightarrow \sigma[20/Y][0/Y]}{\langle \text{while}(Y)\{Y=(Y-20);\}, \sigma[20/Y] \rangle \rightarrow \sigma[20/Y][0/Y]}}$$

By the above derivation, executing the statement `while(Y){Y=(Y-20);}` in the state σ terminates in the following state.

$$\sigma[20/Y][0/Y] = \{(X, 10), (Y, 0), (Z, 30)\}$$