A solution for Exercise 5

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Exercise Calculate $P_3(x)$ from the general form of Legendre polynomials.

$$P_n(x) = \frac{1}{2^n n!}\frac{\mathrm{d}^n}{\mathrm{d}x^n}(x^2-1)^n$$

Solution

$$\begin{aligned}
P_3(x) &= \frac{1}{2^3\cdot 3!}\frac{\mathrm{d}^3}{\mathrm{d}x^3}(x^2-1)^3 \\
&= \frac{1}{8\cdot 6}\frac{\mathrm{d}^3}{\mathrm{d}x^3}(x^6-3x^4+3x^2-1) \\
&= \frac{1}{8\cdot 6}\frac{\mathrm{d}^2}{\mathrm{d}x^2}(6x^5-12x^3+6x) \\
&= \frac{1}{8\cdot 6}\frac{\mathrm{d}}{\mathrm{d}x}(30x^4-36x^2+6) \\
&= \frac{1}{8\cdot 6}(120x^3-72x) \\
&= \frac{1}{8}(20x^3-12x) \\
&= \frac{1}{2}(5x^3-3x)
\end{aligned}$$