| (Course Code) | Applied Mathematics | Isao Sasano |
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| College | College of Engineering |
| Department | Department of Information Science and Engineering |
| Grade | 2nd Year Students |
| Semester | First Semester |
| Credit | 2 |
| Course Type | Elective |
| Course Classification | Specialty |
| Mode of Delivery | Lecture |

## Course Outline

Discrete Fourier transform (DFT) is used for processing sounds and graphics in digital computers. This lecture aims at being able to do Fourier series expansion, which forms the basis for DFT. As an introduction to Fourier series expansion we illustrate the least-square method and the orthogonal function expansion. Fourier series expansion is an instance of the orthogonal function expansion. Understanding Fourier series expansion forms the basis for understanding Fourier transform and DFT, which are topics covered in lectures of signal processing.

## Achievement Objectives

1. Understanding the least-square method and being able to approximate given sequences of data or functions by linear functions or quadratic functions
2. Understanding orthogonal functions and being able to do the orthogonal function expansion for given functions by some given set of orthogonal functions
3. Understanding Gram-Schmidt orthogonalisation, which is a method (algorithm) for orthogonalise a set of vectors in an inner product space, and being able to construct an orthogonal set of functions from a given set of functions.
4. Being able to do Fourier series expansion, which is an important instance of the orthogonal function expansion.

## Course Plan

| 【Course Plan】 | 【Assignment (including preparation and review) 】 |
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| 1. Introduction and the least-square method (1)<br>● Approximation of sequences of data in linear functions | Read Section 20.5 of the reference book. |
| 2. The least-square method (2)<br>● Approximation of sequences of data in quadratic functions | Example 2 in Section 20.5 of the reference book |
| 3. The least-square method (3)<br>● Approximation of sequences of data in linear combination of some fixed set of functions | It is not treated in the reference book. |
| 4. The least-square method (4)<br>● Approximation of functions in linear combination of some fixed set of functions | Confer Problem 16 in Section 20.5 of the reference book. |
| 5. The least-square method (5) and the orthogonal function expansion(1)<br>● Approximation of column vectors<br>● Approximation of functions in linear combination of some fixed set of orthogonal functions<br>● An orthogonal set of functions --- Legendre polynomials | Read Section 5.7 and 5.8 of the reference book.<br>Confer Example 2 in Section 5.8 for Legendre polynomials.<br>Confer Section 7.1 for column vectors. |
| 6. The orthogonal function expansion (2)<br>● An orthogonal set of functions --- Trigonometric functions<br>● The orthogonal function expansion | Read Section 5.8 of the reference book.<br>Confer Appendix 3 for formulae about trigonometric functions. |
| 7. The orthogonal function expansion (3)<br>● An example of the orthogonal function expansion --- Fourier series expansion<br>● Orthogonal set of functions with a weight function<br>● An example --- Chebyshev polynomials | Read Section 11.1 for Fourier series expansion.<br>Confer Problem 20 in Section 5.7 for Chebyshev polynomials |
| 8. Mid-term examination and explanation of the answers<br>● Paper-and-pencil test for checking the understanding of the contents of the lectures from the first to the eighth | Review the contents of all the lectures until the last one. |
| 9. The orghogonal function expansion (4)<br>● Examples --- Hermite polynomials and Laguerre polynomials | Confer Problem 18 in Section 5.8 for Hermite polynomials.<br>Confer Example 2 in Section 5.8 for Legendre polynomials. |
| 10. The orthogonal function expansion (5)<br>● The orthogonal function expansion in Chebyshev, Hermite, and Laguerre polynomials<br>● Inner product spaces<br>● An inner product space --- n-dimensional Euclidean space | Read Section 5.7 and 5.8 of the reference book for the orthogonal function expansion.<br>Read Section 7.9 for the inner product spaces. |

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| | Confer Example 3 in Section 7.9 for the n-dimensional Euclidean space. |
| 11. The orthogonal function expansion (6)<br>● Cauchy-Schwarz inequality<br>● Triangle inequality<br>● Orthonormal basis | Read Section 7.9 for Cauchy-Schwarz inequality and Triangle inequality.<br>Read Section 5.7 for the definition of orthonormality<br>Confer Section 7.4 for basis. |
| 12. The orthogonal function expansion (7)<br>● Orthogonal projection<br>● Orthogonal basis<br>● Gram-Schmidt orthogonalisation | Read Section 9.2 for projections.<br>Gram-Schmidt orthogonalisation is not treated in the reference book. Consult some linear algebra textbook. |
| 13. The orthogonal function expansion (8)<br>● An example of Gram-Schmidt orthogonalisation | Gram-Schmidt orthogonalisation is not treated in the referece book. Consult some linear algebra textbook. |
| 14. The orthogonal function expansion (9)<br>● Obtaining Legendre polynomials by Gram-Schmidt orthogonalisation | Gram-Schmidt orthogonalisation is not treated in the reference book. Consult some linear algebra textbook. |
| 15. Final examination and explanation of the answers<br>● Paper-and-pencil test for checking the understanding of the contents of the lectures from the first to the fourteenth | Review the contents of all lectures |

## Evaluation Method and Criteria

Mid-term exam is evaluated on a 40-point scale, final exam a 50-point, and reports a 10-point. When the mid-term exam is M point, the final exam F point, and the repots R point, the overall score is R+M+F*(100-(R+M))/50.

## Textbooks and Reference Materials

A reference book is:
Erwin Kreyszig, Advanced Engineering Mathematics, John Wiley & Sons Inc; 9th International edition, 2006.
The tenth edition was published in 2011 and it may be equally fine.

Note that I do not use this book as a textbook. Note also that this book is thick and covers topics much more than this lecture covers.

**Pre-Course Preparation**

Basic knowledge of linear algebra and analysis

**Office Hours, Contact Method**

Before and after each lecture or any time agreed on by email

**Relevance to Environmental Education**

None